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Issues Affecting the Calculated Value of Life

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I. Introduction

The first step in estimating the value of life is, generally, to estimate the trade-off individuals are willing to make between the risk of death and compensation. In wage-risk studies the estimate of the trade-off takes the form of a slope coefficient on the risk of death variable from regression analysis. The estimated slope coefficient is then used to estimate the value of life. This paper addresses two separate but related issues that affect the calculated value of life. One issue is how the estimate of the slope coefficient may vary depending on the base amount of risk in the observations used for its estimate. This issue is addressed by determining the shape of the indifference curve between compensation and the risk of death. The other issue is the definition of the value of life. Two proofs are presented. One proof shows that the calculated value of life will vary according to the base amount of risk involved in the study. The other shows how the calculated value varies according to how the value of life is defined. An economist who uses the value of life studies should be aware of these influences.

In wage-risk studies the observations used to estimate the slope coefficient on the risk of death are usually gathered in an area where the absolute amount of risk is, say, from .01% to .1%. This amount of risk is small considering that the risk of death spans from 0.0% to 100%. Thus, it is certainly possible that the slope coefficient (and therefore the estimated value of life) would be significantly different if the observations used to estimate it came from a different area of the risk spectrum. This paper contains a proof which shows that for a risk averse individual, the slope coefficient will increase as the base amount of risk increases. The proof is accomplished by determining the shape of the indifference curve between compensation and the risk of death and is supported by evidence found in the literature.

The second issue is the definition of the value of life. The first definition considers how much a person would willingly sacrifice in order to avoid a 100% risk of death. The value of life is estimated by combining the estimated slope coefficient with the indifference curve between compensation and the risk of death. The results of this paper show that assuming the indifference curve is linear, which is the same as assuming the slope coefficient remains constant, leads to a conservative estimate of the amount an individual would sacrifice to avoid a 100% risk of death.

The value of life can also be defined as the amount an individual would pay to eliminate the risk of death. The amount is estimated by using the estimated slope coefficient and the utility of risk avoidance. The proofs show that the two

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methods, the indifference curve method and the utility of risk avoidance method, result in vastly different estimated values of life. A discussion of the reasoning implied by each method follows the proofs.

Section II begins by placing the discussion of the slope coefficient in terms of an indifference curve between the risk of death and compensation. Section II also contains empirical evidence found in the literature concerning the shape of the indifference curve. The section concludes with a proof of the shape of the indifference curve. Section III discusses alternate definitions of the value of life and their effect on the calculated value of life. Section IV contains the conclusions.

II. The Estimate of the Slope Coefficient

Wage-risk studies use the estimated slope coefficient on the risk of death variable to estimate the value of life. A change in the estimated slope coefficient will have a direct influence on the estimated value of life. The slope coefficient is an estimate of the slope of the indifference curve in the area of the indifference curve from which the observations came. For example, if a slope coefficient, estimated from observations gathered in the area where the risk of death was 5/10,000, showed that individuals were willing to trade $400 for a *change* in the risk of death of 1/10,000, then the slope of the indifference curve at the point where the risk of death is 5/10,000 would reflect this trade-off. Only if the indifference curve is linear would the estimated slope coefficient be invariant with respect to the base risk involved with observations. Line OA of Figure 1 illustrates the linear indifference curve.

Viscusi and Evans (1990) provide empirical evidence on the shape of the indifference curve. First they point out that one cannot make any inferences about the shape of the indifference curve when observations to estimate the slope are gathered in only one area of risk. They then estimate the slope of the indifference curve at two points by using observations from two distinct areas of risk for the same individuals. This allows them to estimate the shape of the indifference curve. Their empirical results show that the 'additional compensation required to accept an increase in risk is greater for high base risk'(p. 368). The empirical results correspond to the indifference curve B in Figure 1.

To provide a proof of the shape of the indifference curve, we begin with a utility function that has two arguments, life and compensation. From this utility function we derive the indifference curve for a given level of utility. Then we show that for a risk averse individual, the indifference curve has the shape of curve B in Figure 1. The proof is accomplished by showing that the first and second derivatives of the indifference curve are greater than zero.[1]

The utility function used here is a slightly modified version of the utility function found in Ulph (1982). Begin with the utility function

$$U(A, C) = (g + mA)^{p}C^{q} \tag{1}$$

Where A has a value of 1 if the individual is alive and 0 otherwise; C is the

[1] For illustrative purposes, the proof given in this paper utilizes a specific functional form for the utility mapping. See Weinstein, Shepard and Pliskin (1980) for a proof which uses a more general functional form.

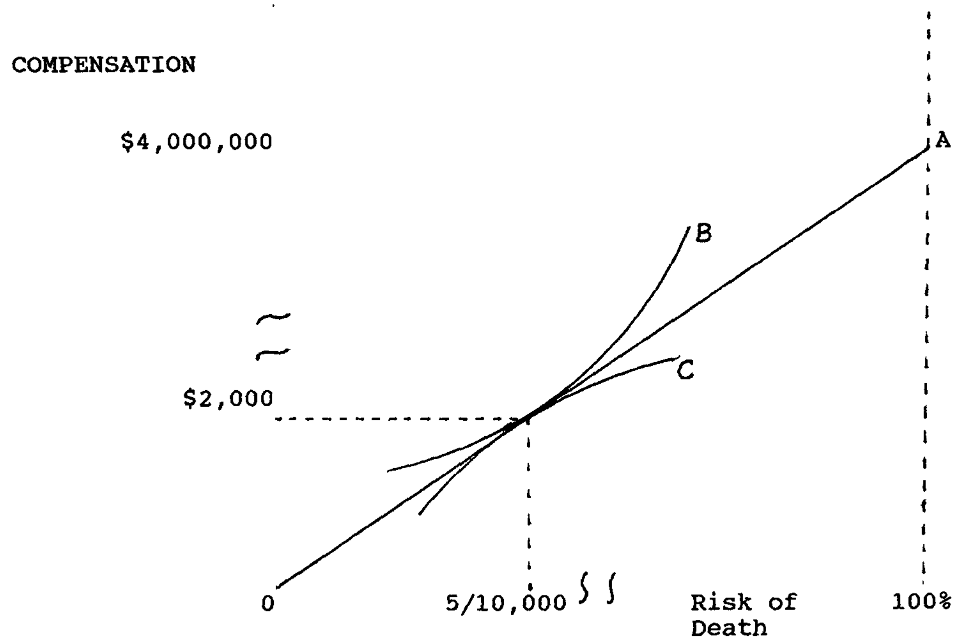

Figure 1

amount of compensation that the individual receives; g is a constant equal to or greater than 0; m is a constant greater than 0; p is a constant greater than 0; q is a constant greater than 0 and less than 1.

This utility function is extremely innocuous. Constraining q to be between 0 and 1 means that an additional unit of C is positively valued but not valued as greatly as the preceding unit. This implies that the individual is risk averse. The amount of utility the individual receives from C depends upon whether the individual is alive; an individual receives greater utility from a given amount of C if alive as opposed to dead. If g is 0, no utility will be derived from C when the individual is dead.

The ex ante combinations of risk of death and compensation that yield a constant level of utility are given by the equation

2) $$(1 - t)(g + m)^p C^q + t(g)^p C^q = k$$

Where k is a constant level of utility; t is the risk of death, $0 \geq t \geq 1$. Solving equation 2) for C gives

3) $$C = (k/((1 - t)\ (g + m)^p + t(g)^p))^{1/q}$$

The first derivative of C with respect to t is

4) $$dC/dt = ((1/q)(k/((1 - t)\ (g + m)^p + t(g)^p))^{(1/q)-1}) \\ ((-k(-(g + m)^p + g^p))\ /\ ((1 - t)\ (g + m)^p + t(g)^p)^2)$$

The second derivative of C with respect to t is

5) $$d^2C/dt^2 = (((1/q)\ -\ 1)(1/q)(k/((1 - t)(g + m)^p + t(g)^p))^{(1/q)-2})$$

$$\cdot((-k(-(g+m)^p+g^p))/((1-t)(g+m)^p+t(g)^p)^2)$$
$$\cdot((-k(-(g+m)^p+g^p))/((1-t)(g+m)^p+t(g)^p)^2)$$
$$+((1/q)(k/((1-t)(g+m)^p+t(g)^p))^{(1/q)-1})$$
$$\cdot((-(-k(-(g+m)^p+g^p))2((1-t)(g+m)^p+t(g)^p))$$
$$\cdot(-(g+m)^p+g^p))/((1-t)(g+m)^p+t(g)^p)^3)$$

Inspection shows that equations 4) and 5) are necessarily positive. This proves that, for a risk averse individual, the indifference curve between risk of death and compensation has the shape of curve B in Figure 1.

This section has shown that the estimated slope coefficient on the risk variable depends upon the base amount of risk in the observations used to estimate the coefficient. Thus, it is possible for a researcher to obtain different values for the slope coefficient and, thereby, different amounts for the calculated value of life, by varying the base amount of risk in the observations. One may argue that the most relevant area from which observations are gathered is that in which individuals generally make decisions, as is the case in most wage-risk studies. It is, however, incumbent upon an economist who uses the value of life studies to know where on the risk spectrum the observations to estimate the slope coefficient originated.

III. Definitions of the Value of Life

Different definitions of the value of life result in vastly different estimates of its value. One definition of the value of life is the amount required to make an individual indifferent between that amount and a 100% risk of death. This amount is estimated by combining the slope coefficient with the indifference curve between compensation and risk of death. From Figure 1 we see that if the slope coefficient is combined with a linear indifference curve, the individual in this example is assumed to have a value of life of $4,000,000. That is, the individual would be indifferent between $4,000,000 and 100% probability of death. Chestnut and Violette (1990) recognize that using a linear indifference curve to calculate the value of life (when the value of life is defined as the amount where an individual is indifferent between the amount and 100% probability of death) may not be legitimate. They state:

> That we can take the change in income and multiply it by the change in risk of death and come up with a dollar per statistical life number does not mean that this is the amount that would compensate the individual or anyone else for that individual's death. Violette and Chestnut (1983) have suggested that it would be reasonable that this dollar per statistical life number would vary with the level of risk involved, but this has not been firmly established empirically. The presumption is that the necessary compensation will increase more rapidly than the acceptable risk and will approach infinity as the risk approaches 100 percent (p. 83).

Chestnut and Violette believe that a linear extrapolation of the slope coefficient may not adequately compensate an individual for a 100% probability of death. The proof provided above concerning the shape of the indifference curve shows that they are correct in their belief.

Since OA is linear, the point of intersection between the tangent to the line at point ($2,000, 5/10,000) and the vertical line at the risk of death of 100% coincides with the point of intersection between the indifference curve and the vertical line. The coincident point is at $4,000,000. Which point of intersection defines the value of life is inconsequential if the indifference curve is linear as both points of intersection coincide. When the indifference curve is not linear, and we have shown that it is not, the two points of intersection will, of course, not coincide.

In order to avoid confusion we refer to the point of intersection between the vertical line at the 100% risk of death and the line tangent to the indifference curve at the point where the observations were centered, which is $4,000,000 in our example, as the ex ante value of life. We refer to the point where the indifference curve intersects the vertical line at the 100% risk of death as the ex post value of life.

Thus, the calculated amount of ex ante compensation required depends on the base amount of risk used in the study. And, the ex post amount required for compensation will always be underestimated if it is calculated by using a linear extrapolation of the estimated slope coefficient.

The second method of calculating the value of life defines it as the amount of money an individual would pay to eliminate the risk of death. This is distinct from the trade-off an individual would be willing to make between compensation and 100% probability of death. Calculating the value of life using this method involves combining the estimated slope coefficient on the risk of death with the marginally declining value of risk avoidance.

Using this marginally declining value of risk avoidance to define the value of life results in a value that is less than that calculated with either the indifference curve A or B in Figure 1. Viscusi states:

> Individuals may be willing to pay $600 to produce a reduction in their lifetime risk of 1/10,000, but this does not mean that we can extrapolate from this rate of tradeoff to assess how much individuals would be willing to pay for more substantial risk reductions. The amount that people will be willing to pay to produce a risk reduction of 1/1,000 will for example be less than $6,000 in general, and carrying this process further the amount they would pay to purchase a 1/10 reduction in death risks would be less than $600,000 (1990, p. 10).

Carrying this statement to its logical extreme implies that an individual would be willing to accept a 100% risk of death for some amount less than $6,000,000. Although his statement may initially seem to infer the use of indifference curve labeled C in Figure 1 to obtain the value of life, that is not the case. Viscusi refers to the willingness to pay for different amounts of change of risk (the decreasing marginal value of risk reduction) rather than the willingness to pay for constant amounts of change of risk measured at different areas of risk. He combines the estimated slope coefficient with the declining marginal value of risk reduction rather than with the indifference curve to calculate the value of life. Figure 2 shows the shape of the expected utility of the risk reduction curve. Appendix 1 contains a proof of the shape of the curve in Figure 2. Thus, given an estimated slope coefficient, the definition of the value of life the researcher uses will affect its calculated value. Defining the value of life as the amount an individual would pay for risk reduction will result in a calculated value that is

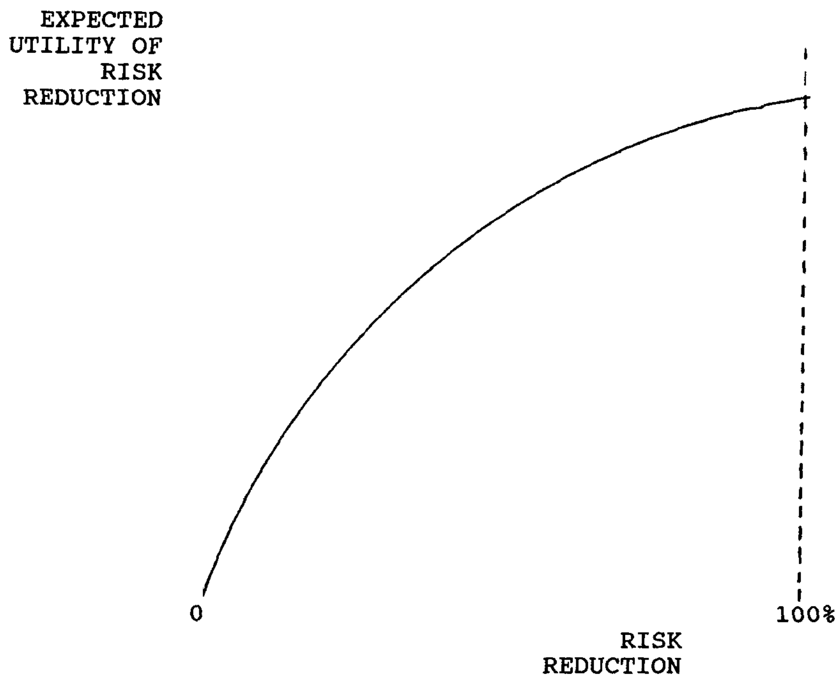

Figure 2

less than either the ex ante or ex post amounts defined by using the indifference curve.

Viscusi's definition of the value of life does not attempt to answer the question of the amount of money necessary to make an individual indifferent between that amount and a particular probability of death. Rather, the value of life is defined by the amount of money an individual would pay to eliminate the risk of death; it may be thought of as a demand curve where an individual purchases units of risk reduction. When Viscusi's definition is used, the income effect is a factor that influences the value of life. The question Viscusi poses is, 'How can individuals value their lives from a standpoint of deterrence by so much more than their total lifetime resources?'(1990, p.10). Certainly, we would anticipate that an individual's level of income would have a large effect upon their willingness to purchase more and more units of risk avoidance. This may be contrasted with defining the value of life in terms of the indifference curve. An individual can be indifferent between $100 million and 99% risk of death even though the individual does not have $100 million. The calculated value of life is a function of the level of income to a much greater extent when the risk reduction definition is used as opposed to when the indifference curve definition is used.

IV. Conclusion

The value of life is often calculated by using the estimated slope coefficient on the risk of death variable in regression analysis. The slope coefficient reports the trade-off an individual is willing to make between risk and compensation. This paper contains a proof that shows for risk averse individuals the slope coefficient increases as the base amount of risk involved increases. Therefore, the ex ante calculated value of life will depend upon the amount of risk in the observations used to estimate the coefficient; the greater the base risk in the observations the greater the ex ante calculated value of life. Another implication of the proof of the changing slope coefficient is that the ex post value of life (the actual point of intersection between the indifference curve and the vertical line at the 100% risk level), which is the amount that an individual would willingly trade for 100% probability of death, will always be underestimated when calculated using a linear extrapolation of the slope estimate.

The calculated value of life will vary depending on how it is defined. This paper considered two definitions of the value of life. One definition uses the indifference curve between compensation and the risk of death. The other uses the utility of risk reduction. By proving that the utility of risk reduction is marginally declining, we show that the calculated value of life based on it is less than the calculated value of life based on the indifference curve. Both issues discussed in this paper have a potentially large effect on the calculated value of life.

Appendix

To show the declining marginal expected utility of risk reduction we begin with the utility function of equation 1) (repeated here as 1A)).

1A) $$U(A, C) = (g + mA)^p C^q$$

Taking expected values

2A) $$E(U) = E((g + mA)^p C^q)$$
$$= (g + mE(A))^p C^q$$

Recall that $A = 1$ when the individual is alive and $A = 0$ otherwise. Then

3A) $$E(A) = r(1) + (1 - r)(0)$$

Where $0.0\% \leq r \leq 100\%$ is the amount of risk reduction obtained. Then,

4A) $$E(U(r)) = (g + mr)^p C^q$$

5A) $$dE/dr = p(g + mr)^{p-1} C^q mr$$

6A) $$d^2E/dr^2 = (p - 1)p(g + mr)^{p-1} C^q (mr)^2$$

Inspection shows that equation 5A) is greater than zero. And, requiring p to be less than 1, which implies that the individual is risk averse, constrains equation 6A) to be less than zero. This proves that, for a risk averse individual, risk reduction has decreasing marginal expected utility.

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